Heat Transfer in a Fluidized Bed of Small Particles

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Experimental data for particle to fluid heat transfer in fixed and fluidized beds of small particles at low Reynolds numbers show abnormally low Nusselt numbers when the total particle surface is assumed to be the heat transfer surface. Kunii and Levenspiel (1969) present a summary of these data. Two explanations have been proposed for this apparent anomaly. Richardson and Szekely (1961) suggested gas back mixing and Kunii and Suzuki (1967) proposed a channeling model for molecular diffusion within clusters of particles. They derived an equation for heat transfer in a channeled bed based upon the total surface area of the particles.

$$h_p = \frac{C_p \rho u_s \varphi}{6(1-\epsilon)\xi}(1-\psi) \qquad (1)$$

Analysis of residence time models indicates that $(1 - \psi) \approx 1$ for low Peclet numbers.

PARTICLE CLUSTER HEAT TRANSFER

A prior paper (Hughmark, 1972) showed that the Carman channel model applies for the laminar regime of packed and fluidized beds. The void fraction of a bed of discrete particles is represented by

$$\frac{\epsilon^3}{1-\epsilon} = \frac{181\, u_s \mu}{g(\rho_s - \rho) D_p^2} \qquad (2)$$

The concept of particle clusters can be represented by consideration of these clusters as particles to which the Carman channel model must also apply. Thus a fluidized bed of small particles may be represented as a fixed bed of larger particles corresponding to the clusters as larger particles. The void fraction of the bed of clusters ϵ_0 with diameter d then has a pressure drop across the bed that corresponds to the void fraction of the discrete particle bed.

$$\frac{\epsilon_0^3}{1-\epsilon_0} = \frac{181\, u_s \mu}{g(\rho_s - \rho)(1-\epsilon) d^2} \qquad (3)$$

Equations (2) and (3) then give the relationship between the particle sizes and void fractions

$$d = \left(\frac{\epsilon}{\epsilon_0}\right)^{3/2} \frac{(1-\epsilon_0)}{(1-\epsilon)} D_p \qquad (4)$$

The ξ term of the Kunii and Suzuki model is equivalent to d/D_p. Substitution in Equation (1) with $\psi = 0$ yields

$$h_p = \frac{C_p \rho u_s}{6} \frac{\epsilon_0^{3/2}}{\epsilon^{3/2}(1-\epsilon_0)} \qquad (5)$$

Equation (5) provides the heat transfer coefficient based upon the total particle surface. The coefficient based upon the surface of a spherical cluster of particles can be derived as

$$h_{pc} = h_p\, \epsilon^{3/2}/\epsilon_0^{3/2} \qquad (6)$$

Substitution of Equation (5) yields

$$h_{pc} = \frac{C_p \rho u_s}{6(1-\epsilon_0)} \qquad (7)$$

Equations (5) and (7) provide for heat transfer within the cluster. Total heat transfer for the bed could be expected to correspond to the additive resistances from transfer within the particle clusters and heat transfer from the particle clusters to the channels.

CHANNEL HEAT TRANSFER

The prior paper showed that laminar flow mass transfer in fixed beds could be represented by laminar flow mass transfer with the channel model. Experimental data are correlated by the equation

$$\frac{kD_e}{D} = (N_{Re} N_{Sc})^{1/3} \qquad (8)$$

Combination of Equation (4), the heat transfer analogy of Equation (8), and the channel equivalent diameter

$$D_e = 2/3 \frac{\epsilon^{3/2}}{\epsilon_0^{3/2}(1-\epsilon)} D_p$$

provides the channel heat transfer coefficient in terms of the particle diameter

$$h_c = 1.47 \rho C_p \frac{\alpha^{2/3}}{D_p^{2/3}} \frac{(1-\epsilon)^{2/3}}{\epsilon} u_s^{1/3} \qquad (9)$$

Additive resistances from Equations (7) and (9) provide the total heat transfer coefficient based upon the channel area. Equation (6) provides transition to the total surface area.

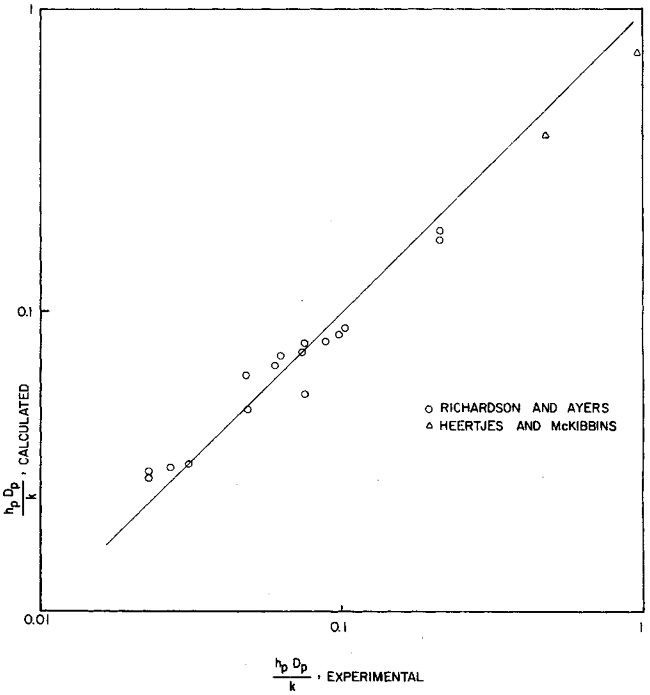

Fig. 1. Heat transfer coefficients.

COMPARISON WITH EXPERIMENTAL DATA

The data of Richardson and Ayers (1959) for glass beads and air and Heertjes and McKibbins (1956) for silica gel and air were compared with calculated heat transfer coefficients for the total surface area. A value of $\epsilon_0 = 0.38$ was used as a typical void fraction for a fixed bed. Figure 1 shows this comparison with reasonable agreement between calculated and experimental coefficients.

NOTATION

C_p = specific heat
D = diffusivity
D_e = equivalent diameter of channel
D_p = particle diameter
d = particle cluster diameter
g = acceleration due to gravity
h_c = channel heat transfer coefficient
h_p = heat transfer coefficient based on total particle surface
h_{pc} = particle cluster coefficient based on channel area
k = mass transfer coefficient
N_{Re} = $D_e u_e/\nu$, Reynolds number
N_{Sc} = Schmidt number
u_e = actual velocity in packing channels
u_s = superficial velocity

Greek Letters

α = thermal diffusivity of fluid
ϵ = overall bed void fraction
ϵ_0 = void fraction of bed for particle clusters
ξ = ratio of average channelling length to particle diameter
μ = fluid viscosity
ν = kinematic viscosity of fluid
ρ = fluid density
ρ_s = solids density
ψ = dimensionless temperature or concentration
φ = particle shape factor

LITERATURE CITED

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Manuscript received January 19, 1973, and accepted January 22, 1973.

On Kinetic Behavior at High Enzyme Concentrations

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Currently there are considerable interest and activities in potential applications of enzymes in various forms as industrial catalysts, some of which involve a large concentration of enzymes. For details one may refer to the review paper by Carbonell and Kostin (1972). For enzyme kinetics at high enzyme concentrations Carbonell and Kostin also refer to the work of Cha (1970), who compared what he called the true rate equation with a number of approximate rate equations which includes among others the Michaelis-Menten equation.

The purpose of this note is twofold: (1) to show that the rate equation which was developed by Reiner (1969) and subsequently used by Cha (1970) as the true rate is based on a misconception and is entirely equivalent to the Michaelis-Menten equation in the form of present day use (or more correctly Briggs-Haldane) and hence the comparisons made by Cha (1970) are incorrect, and (2) to bring attention to the fact that when the total enzyme concentration is high relative to the initial substrate concentration the pseudo-steady state (PSS) assumption which leads to the Michaelis-Menten (or Briggs-Haldane) equation may not hold, and therefore a valid comparison would be with the true rate which must be computed without the PSS assumption. This comparison is made using the data reported in the literature on the hydrolysis of acetyl-L-phenyl-alanine ethyl ether by chymotrypsin (Gutfreund and Hammond, 1959) at various concentration ratios of enzyme to the initial substrate. It is very important to fully realize the limitations imposed by the PSS assumption as currently there are being developed many potential applications involving use of concentrated enzymes.

RATE EQUATIONS

The reaction under consideration is the simplest one substrate-one enzyme reaction in a batch reactor,

$$E + S \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} ES \overset{k_2}{\rightarrow} E + P \tag{1}$$

Since the mechanism involves four species, four rate equations can be written,

$$d(E)/dt = -k_1(E)(S) + (k_{-1} + k_2)(ES) \tag{2}$$

$$d(S)/dt = -k_1(E)(S) + k_{-1}(ES) \tag{3}$$

$$d(ES)/dt = k_1(E)(S) - (k_{-1} + k_2)(ES) \tag{4}$$

$$d(P)/dt = k_2(ES) \tag{5}$$

with the presumed initial conditions $(E)_{t=0} = (E_t)$, $(S)_{t=0} = (S_0)$ and $(ES)_{t=0} = (P)_{t=0} = 0$.
There are two conservation equations